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CSIS 213-3941

Exam 2 Part 2

**Question 4**

The following sequence is deﬁned recursively. Use iteration to guess an explicit formula for the sequence.

*ek*= 4*ek*−1 + 5, for all integers *k*≥ 1

*e*0  = 2

 After that, use mathematical induction to verify the correctness of the formula you obtained.

**Proof:**

*ek*= 4*ek*−1 + 5

*e*0  = 2

*e1*= 4*e1*−1 + 5 = 4(2) + 5 = 8 + 5

*e2*= 4*e2*−1 + 5 = 4(13) + 5 = 57

*e3*= 4*e3*−1 + 5 = 4(57) + 5 = 4(8 + 5 \* 3)

*Explicit Formula:* en = 4(e0 + 5n) for n≥ 0

**Proof of Correctness:**

*Show that P(*1*) is true*

To establish P(1), we must show that

*e1*= 4*e1*−1 + 5

ek = 20k + 32

But the left-hand side of P(1) is

*e1 = 20(1) + 32 = 52*

and the right hand side of P(1) is

4*e1*−1 + 5 = 4(2) + 5 = 8 + 5 = 13

Thus the two sides of P(1) equal the same quantity and hence P(1) is true.

*Show that for all integers k >= 1, if P(k) is true then P(k + 1) is also true*

*ek*= 4*ek*−1 + 5

We must show that

*ek + 1*= 4*e(k* + 1)−1 + 5

But the left hand side of P(k + 1) is

*ek + 1*= 4*e(k* + 1)−1 + 5

= 4ek + 5

= 4(4*ek*−1 + 5) + 5

= 16*ek*−1  + 25